Universal relaxation of pinned density waves in Condensed Matter and Holography

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- Rich dialogue between holography and effective theories: eg KSS bound, anomalies.
$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi}$$
- First established in the context of asymptotically AdS black branes: Schwarzschild, RN, etc.
- Sharp definition in the context of **relativistic hydrodynamics**: η quantifies the diffusion of transverse momentum

$$G^R_{P_\perp P_\perp}(\omega, q) = \frac{\chi_{PP} D_\perp q^2}{i\omega - D_\perp q^2}\,, \qquad D_\perp = \frac{\eta}{\chi_{PP}}$$

- Weaker bound $\eta > 0$ for **positivity of entropy production**.
- Less symmetric cases? This talk: **broken translations in 2d isotropic Wigner crystals**.

- Talk mostly based on
  - *'Theory of the collective magnetophonon resonance and melting of the field-induced Wigner solid'* [arXiv:1904.04872], PRB'19, with L. Delacrétaz, S. Hartnoll and A. Karlsson (DHK)
  - *'Universal relaxation in a holographic density wave phase'* [arXiv:1812.08118], to appear in PRL, with A. Amoretti, D. Areán and D. Musso (AAM)
  - *'Diffusion and universal relaxation of holographic phonons'* [arXiv:1904.11445], JHEP'19, AAM
- But see also
  - *'Bad Metals from Density Waves'* [arXiv: 1612.04381], Scipost'17, DHK
  - *'Theory of hydrodynamic transport in fluctuating electronic charge density wave states'* [arXiv:1702.05104], PRB'17, DHK
  - *'DC resistivity of quantum critical, charge density wave states from gauge-gravity duality'* [arXiv:1712.07994], PRL'18, AAM
  - *'Effective holographic theory of charge density waves'* [arXiv:1711.06610], PRD'18, AAM
  - *'Gapless and gapped holographic phonons'* [arXiv:1910.11330], AAM

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Related recent work:

- Probe branes: Jarvinen, Jokela, Lippert [arXiv:1408.1397], [arXiv:1612.07323], [arXiv:1708.07837]
- Bianchi VII: Andrade, Baggioli, Krikun, Poovuttikul [arXiv:1708.08306], [arXiv: 1812.08132]
- Massive gravity: Alberte, Ammon, Baggioli, Gray, Grieninger, Jimenez, Pujolas [arXiv:1708.08477], [arXiv:1711.03100], [arXiv:1904.05785], [arXiv:1905.09164], [arXiv:1905.09488]
- Q-lattices: Donos, Martin, Pantelidou, Ziogas [arXiv:1903.05114], [arXiv:1905.00398], [arXiv:1906.03132]
- Inhomogeneous spatially modulated phases: Donos [arXiv:1303.7211], Withers [arXiv:1304.0129,1304.2011,1407.1085], Donos and Gauntlett [arXiv:1512.06861], Andrade, Krikun, Schalm and Zaanen [arXiv:1710.05791], Cai, Li, Wang and Zaanen [arXiv:1706.01470], Cremonini, Li and Ren [arXiv:1612.04385,arXiv:1705.05390], Donos, Gauntlett, Griffin and Ziogas [arXiv:1801.09084], Goutéraux, Jokela and Ponni [arXiv:1803.03089]
- Higher-form global symmetries: Grozdanov and Poovuttikul [1801.03199], Armas and Jain [1908.01175].

- When translations are spontaneously broken, new gapless degrees of freedom are generated: the **Goldstone boson**s (phonons).

- New transport degrees of freedom: **constrained by positivity of entropy production**:

$$\gamma_1^2 \leq \mathrm{Max}\left(\sigma_o \Xi \, , \frac{\sigma_o \Omega}{\chi_{PP}\omega_o^2}\right)$$

- Under certain circumstances, at low temperatures, these new bounds are saturated: **no entropy production**

$$\gamma_1^2 = \mathrm{Max}\left(\sigma_o \Xi \, , \frac{\sigma_o \Omega}{\chi_{PP}\omega_o^2}\right)$$

  - In a holographic toy model of broken translations
  - In 2d electron gases in a large magnetic field (GaAs heterostructures)

- Holographic model: $\Omega/\omega_o^2 = \chi_{PP}\Xi$

- The low energy dynamics of the ordered phase differ from those of the disordered phase by the necessity to include **new gapless degrees of freedom** (the Goldstones).

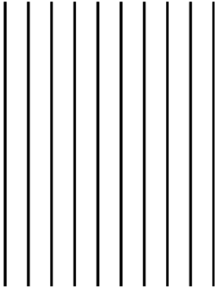
- An important property of Goldstones is that they are **shift-symmetric**: they realize non-linearly the broken symmetry. More concretely, take broken translations along x

$$x \to x + c \quad \Rightarrow \quad \varphi \to \varphi + c$$

Shift symmetry: **only gradient terms** in the effective IR action:

$$f = \frac{1}{2}(K + G)\lambda_{\|}^2 + \frac{1}{2}G\lambda_{\perp}^2 + \dots$$

where $\lambda_{\|} = \nabla \cdot \vec{\varphi}$, $\lambda_{\perp} = \nabla \times \vec{\varphi}$.

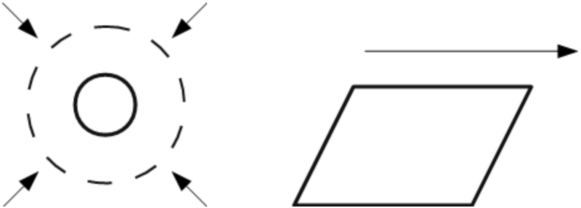


$$f = \frac{1}{2}(K+G)\lambda_{\parallel}^2 + \frac{1}{2}G\lambda_{\perp}^2 + \dots$$

- K and G are the bulk and shear moduli:

$$\chi_{\lambda_{\parallel}\lambda_{\parallel}} \equiv \frac{\delta\lambda_{\parallel}}{\delta s_{\parallel}} = \frac{1}{K+G}, \qquad \chi_{\lambda_{\perp}\lambda_{\perp}} \equiv \frac{\delta\lambda_{\perp}}{\delta s_{\perp}} = \frac{1}{G}$$

They are the static response to bulk compression and shear stress.



- Since π^i is the charge that generates the symmetry, then

$$[\varphi_i(x), \pi_j(y)] = i\delta_{ij}\delta(x-y) + \dots$$

- The effective Hamiltonian contains a term

$$H = \int d^d x \, v_i(x)\pi^i(x) + \dots$$

which leads to the 'Josephson' relations

$$\dot{\lambda}_{\parallel} = \nabla \cdot v + O(\nabla^2), \qquad \dot{\lambda}_{\perp} = \nabla \times v + O(\nabla^2)$$

- At higher order in gradients (+relativistic symmetry), linear, diffusive couplings

$$\dot{\lambda}_{\parallel} = \nabla \cdot v + \gamma_1 T \nabla^2 \left(\frac{\mu}{T}\right) + \xi_{\parallel} \nabla^2 \lambda_{\parallel} + O(\nabla^3),$$

$$\dot{\lambda}_{\perp} = \nabla \times v + \xi_{\perp} \nabla^2 \lambda_{\perp} + O(\nabla^3)$$

- Constitutive relation for the electric current

$$j = \rho v - \sigma_o T \nabla \left(\frac{\mu}{T} \right) - \gamma_1 \nabla \lambda_{\parallel}$$

- Isotropic crystal

$$\frac{\xi_{\parallel}}{K + G} = \frac{\xi_{\perp}}{G} \equiv \Xi$$

- Bound ensuring positivity of entropy

$$\gamma_1^2 \leq \sigma_o \frac{\xi_{\parallel}}{K + G} = \sigma_o \Xi$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{Z(\phi)}{4}F^2 - V(\phi) - Y(\phi)\left(\partial\psi_x^2 + \partial\psi_y^2\right)\right]$$

$$Y(\phi) = \phi^2 + O(\phi^3)\,, \quad Z(\phi) = 1 + O(\phi)\,, \quad V(\phi) = -6 + \phi^2 + O(\phi^3)$$

- Homogeneous generalized Q-lattice Ansatz [Andrade & Withers'13, Donos & Gauntlett'13]: $\psi_i = kx^i$. Breaks Translations$\times$Global shifts to a diagonal U(1).

- UV boundary conditions on ϕ

$$\phi = \lambda r + \phi_v r^2 + \dots$$

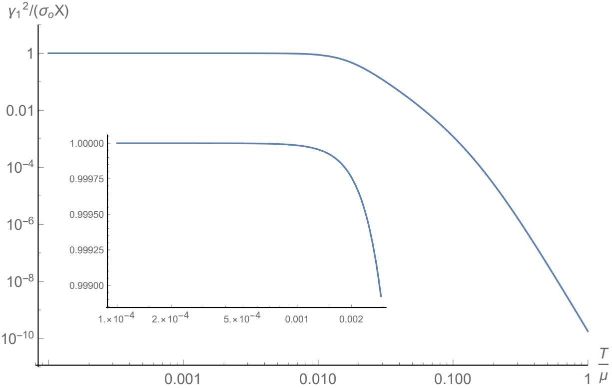
- If $\lambda = 0$, then $\psi_i = kx^i$ is a vev: **spontaneous breaking**.

- If $\lambda \neq 0$, then $\psi_i = kx^i$ is a source: **explicit breaking**.

- But if $\lambda/\mu \ll \phi_v/\mu^2$, **pseudo-spontaneous breaking**.

We compute γ_1, σ_o and Ξ holographically (using radially conserved bulk currents). The positivity of entropy production bound is obeyed

$$\sigma_o \Xi - \gamma_1^2 \geq 0$$



Even **saturates** at low T. Why?

From

$$
\begin{aligned}
G^R_{J\varphi_\parallel}(\omega, q=0) &= \gamma_1 + \frac{i\rho}{\chi_{PP}\,\omega}\,, \\
G^R_{\varphi_\parallel\varphi_\parallel}(\omega, q=0) &= \frac{1}{\chi_{PP}\,\omega^2} - \Xi\frac{i}{\omega}\,,
\end{aligned}
$$

we can write Kubo formulæ

$$
\begin{aligned}
\gamma_1 &= \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,G^R_{J\dot\varphi}(\omega, q=0)\,, \\
\Xi &= \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,G^R_{\dot\varphi\dot\varphi}(\omega, q=0)\,.
\end{aligned}
$$

All we need is a mechanism producing a nonzero $\partial_t\varphi$.

The saturation at low temperatures can be explained by **universal relaxation into the heat current**

$$\Delta H = \int dx \frac{\pi \cdot j_q}{\chi_{PJ_q}} \quad \Rightarrow \quad \dot{\varphi} = \frac{j_q}{\chi_{PJ_q}}$$

$$\gamma_1 = \quad \tfrac{1}{\chi_{PJ_q}} \lim_{\omega\to 0} \tfrac{1}{\omega} \mathrm{Im} G^R_{jj_q}(\omega, q=0) \quad = -\frac{\mu}{\chi_{PJ_q}}\sigma_o \,,$$

$$\Xi = \quad \left(\tfrac{1}{\chi_{PJ_q}}\right)^2 \lim_{\omega\to 0} \tfrac{1}{\omega} \mathrm{Im} G^R_{J_qJ_q}(\omega, q=0) \quad = \left(\frac{\mu}{\chi_{PJ_q}}\right)^2 \sigma_o \,.$$

These values verify $\gamma_1^2 = \sigma_o \Xi$ and match our numerics

Now, break translations **explicitly, weakly**.

The Goldstones become **massive** and **damped**

$$f = \frac{1}{2}(K+G)(\nabla \cdot \varphi)^2 + \frac{1}{2}G(\nabla \times \varphi)^2 + \frac{1}{2}m^2\varphi^2 + \dots$$

$$\dot{\varphi}^i = -\Omega\varphi^i + O(q)$$

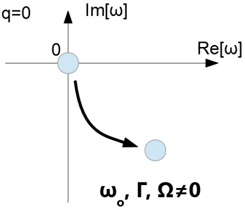
Also relaxes momentum

$$\dot{\pi}^i = -\Gamma\pi^i - Gm^2\varphi^i + \dots$$

All poles are gapped

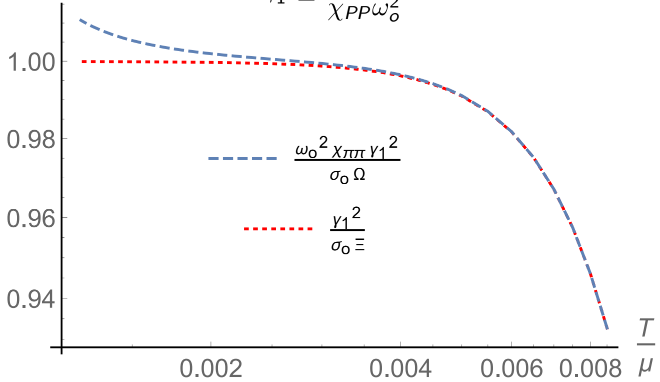
$$(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2 = 0$$

with $\omega_o \equiv m\sqrt{(}G/\chi_{PP})$ the pinning frequency.



- There is another positivity of entropy production bound

$$\gamma_1^2 \leq \frac{\Omega\sigma_o}{\chi_{PP}\omega_o^2}$$



- **Almost saturates**: relaxation of the weakly-gapped phonons into the heat current.
- Violation at low temperature: **breakdown of WC hydro picture** (the phonons become very strongly damped).

- Now turn on a magnetic field: the longitudinal and transverse sound modes hybridize into (gapless) **magnetophonons** and gapped magnetoplasmons.

- Upon turning on disorder, the magnetophonons are pinned at $\omega_o^2/\omega_c \sim O(1/B)$: within hydrodynamics at **large magnetic fields**.

- Write down a similar hydrodynamic theory as before: conservation of charge, Josephson for magnetophonon, constitutive relations, solve and get conductivity.

- Positivity of entropy production bound.

- We want to **break magnetic translations**. When broken generators do not commute, reduction on the number of expected Goldstones [Watanabe & Muruyama'12].
- Under **magnetic translations**, Goldstones $\varphi_i \to \varphi_i + \delta x_i$. Leads to

$$\mathcal{L} = \epsilon^{ij} \varphi_i \dot{\varphi}_j$$

Upon quantizing

$$[\varphi_i(x), \varphi_j(y)] = -i\epsilon_{ij}\delta(x-y)$$

The Goldstones are **not independent fields**!

- From Noether, conserved densities are $\pi^i \sim \epsilon^{ij}\varphi_j$, which leads to the magnetic translation algebra:

$$\Rightarrow [\mathcal{P}_i, \mathcal{P}_j] = -iRB\epsilon_{ij}$$

- The free energy including pinning and leading spatial gradients

$$\mathcal{L} = \epsilon^{ij}\varphi_i\dot{\varphi}_j - \varphi_j\left[\delta^{ij}\omega_{\rm pk} + \left(Kk^ik^j + Gk^2\delta^{ij}\right) + \ldots\right]\varphi_j$$

- Leads to the modes [Fukuyama & Lee'78]

$$\omega(k) = \pm\sqrt{\left(\omega_{\rm pk} + Gk^2\right)\left(\omega_{\rm pk} + (K+G)k^2\right)}$$

$$\begin{cases} \omega_{\rm pk} = 0 & \Rightarrow \quad \omega(k) = \pm k^2 \\ k = 0 & \Rightarrow \quad \omega = \pm\omega_{\rm pk} \end{cases}$$

- Enters relaxation

$$\begin{pmatrix} j^i \\ \varphi^i \end{pmatrix} = \begin{pmatrix} \sigma_o^{ij} & \gamma^{ij} \\ \gamma^{ij} & \Omega^{ij}/\omega_{\rm pk} \end{pmatrix} \begin{pmatrix} E_j \\ s_j - \omega_{\rm pk}\varphi_j \end{pmatrix}$$

$$\sigma_o^{ij} = \sigma_o \delta^{ij} + \sigma_o^H \epsilon^{ij} \,, \gamma^{ij} = \gamma \delta^{ij} + \sqrt{\nu}\epsilon^{ij} \,, \Omega^{ij} = \Omega \delta^{ij} + \omega_{\mathsf{pk}} \epsilon^{ij}$$

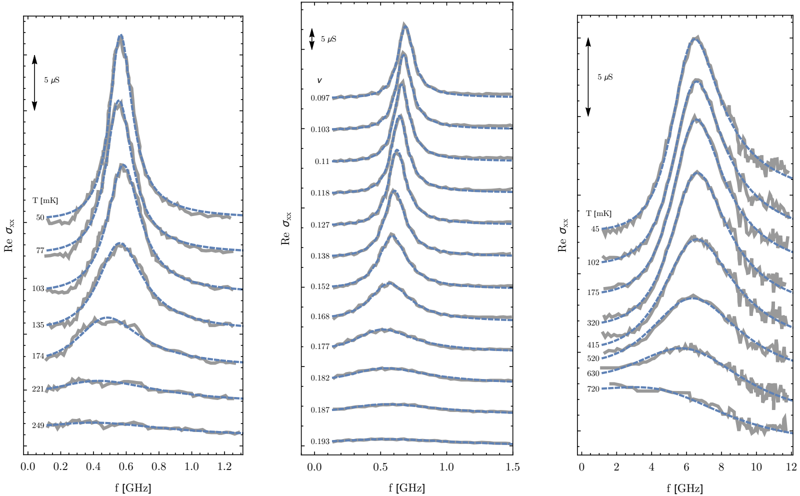
- Conductivity

$$\sigma_{xx}(\omega) = \sigma_o + \nu\, \omega_{\mathsf{pk}} \frac{(1-a^2)(-i\omega + \Omega) - 2a\omega_{\mathsf{pk}}}{(-i\omega+\Omega)^2 + \omega_{\mathsf{pk}}^2} \qquad \nu = \frac{\rho}{B}\,.$$

- **new:** $a \equiv \gamma/\sqrt{\nu}$ asymmetry parameter.
- Positivity of entropy production:

$$\gamma^2 \leq \frac{\sigma_o \Omega}{\omega_{\mathsf{pk}}}$$

Fit to data on GaAs heterojunctions (2DEG) [YP Chen at al, Nature Physics'06], [YP Chen et al, International Journal of Modern Physics B'07], [YP Chen, PhD thesis'05]

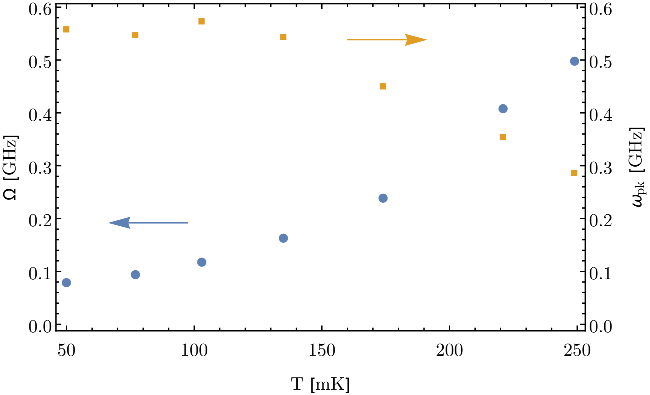


Sample A
Thermal melting

Sample B
Quantum melting

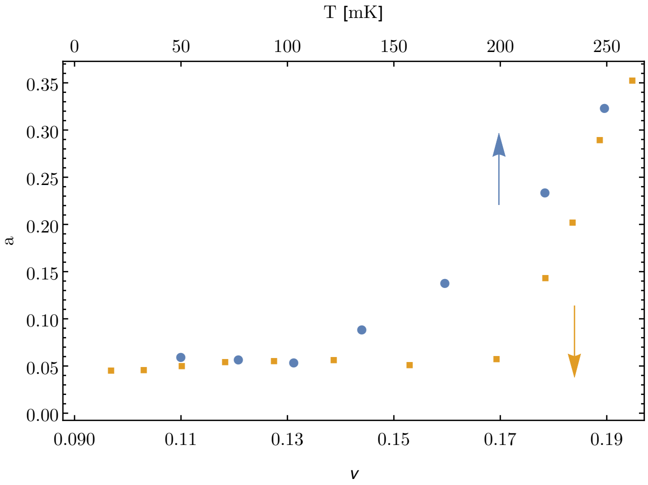
Sample C
Thermal melting
(more disordered)

Ω increases as melting is approached: **shorter-lived magnetophonon**.



sample A

The fits require a **nonzero asymmetry parameter** $a \neq 0$:



samples A and B

- Compute relaxation parameters? Use Kubo formulas

$$\Omega = \omega_{\text{pk}} \lim_{\omega\to 0} \lim_{\Omega,\gamma\to 0} \frac{1}{\omega} \text{Im}\, G^R_{\dot{\varphi}_x\dot{\varphi}_x}(\omega)\,,$$
$$\gamma = \lim_{\omega\to 0} \lim_{\Omega,\gamma\to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{j_x\dot{\varphi}_x}(\omega)\,,$$

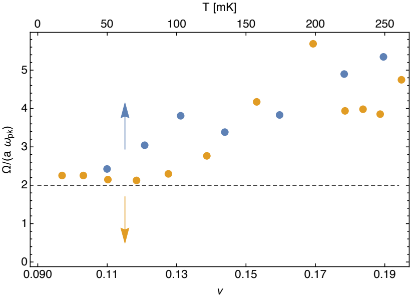
- Now need to compute $\dot{\varphi}$.
  - **Mobile dislocations**

$$\Omega_{\text{vor}} = \frac{2x}{\sigma_{\text{n}}}\nu\omega_{\text{pk}}\,, \quad \gamma_{\text{vor}} = x\sqrt{\nu}\frac{\sigma_{\text{n}}^H}{\sigma_{\text{n}}} \quad \Rightarrow \quad \frac{\Omega}{a\omega_{\text{pk}}} = 2\,.$$

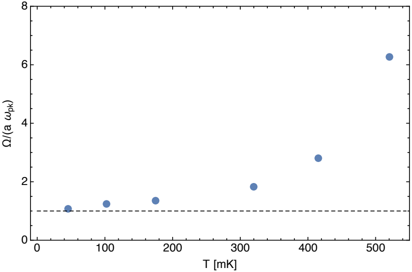
  - **Relaxation into current** $H_{\text{dis}} = \frac{1}{\sqrt{\nu}} \int d^2x \epsilon_{ij}\varphi_i(x) j_j(x)\,.$

$$\Omega_{\text{dis}} = \frac{\omega_{\text{pk}}\sigma_0}{\nu}\,, \quad \gamma_{\text{dis}} = \frac{\sigma_0}{\sqrt{\nu}} \quad \Rightarrow \quad \frac{\Omega}{a\omega_{\text{pk}}} = 1\,.$$

Different microscopic relaxation mechanisms appear to be at play in the different samples:



Samples A and B
mobile dislocations



Sample C (more disordered)
universal dissipation into hydrodynamic currents

- We observed the **saturation** of two bounds on positivity of entropy production

$$\gamma^2 \leq \sigma_o \Xi\,, \qquad \gamma^2 \leq \frac{\sigma_o \Omega}{\chi_{PP}\omega_o^2}$$

- Explained by **universal relaxation** of Goldstones into the **heat current**. Is this generic? See also [Davison, Schalm, Zannen'13], [Lucas'15].

$$\gamma^2 \leq \sigma_o \Xi \,, \qquad \gamma^2 \leq \frac{\sigma_o \Omega}{\chi_{PP}\omega_o^2}$$

- This suggests a relation between the Goldstone relaxation rate and mass should hold at low enough T

$$\Omega = \chi_{PP}\omega_o^2 \Xi$$

- In the holographic model, we can actually show it is a consequence of weak explicit breaking of translations, rather than low temperature. It holds even at higher temperatures, where the entropy bounds are far from saturation.

- Universal contribution to the damping rate? Artefact of the holographic toy model? Other holographic models?

- An argument for universality

$$\partial_t \varphi = v + \Xi \nabla^2 s_\varphi$$

- Spontaneous: $s_\varphi = G k^2 \varphi$
- Pseudo-spontaneous: $s_\varphi = G(k^2 + m^2)\varphi$

$$\partial_t \varphi = v + G\Xi \nabla^2 \varphi + G m^2 \Xi \varphi$$

$$\Rightarrow \Omega = G m^2 \Xi$$

- In which circumstances is this the dominant contribution to the damping rate Ω?