Quantum hydrodynamics for quantum many-body chaos

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Slogan

Hydrodynamics is conventionally defined as in terms of derivative expansion in the regime:

$$\delta L \gg \ell_{\rm relax}, \quad \delta t \gg \tau_{\rm relax}$$

In this talk I will discuss a formulation of fluctuating hydrodynamics applicable to quantum many-body systems in the regime

$$\delta L \sim \ell_{\rm relax} \qquad \delta t \sim \tau_{\rm relax}$$

and use it to predict new manifestations of quantum many-body chaos.

Plan

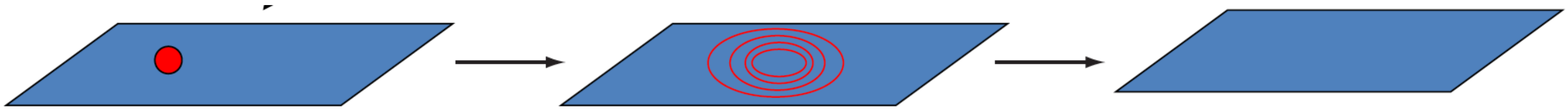
1. Brief review recent progress in understanding quantum many-body chaos

2. Quantum hydrodynamics for quantum many-body chaos

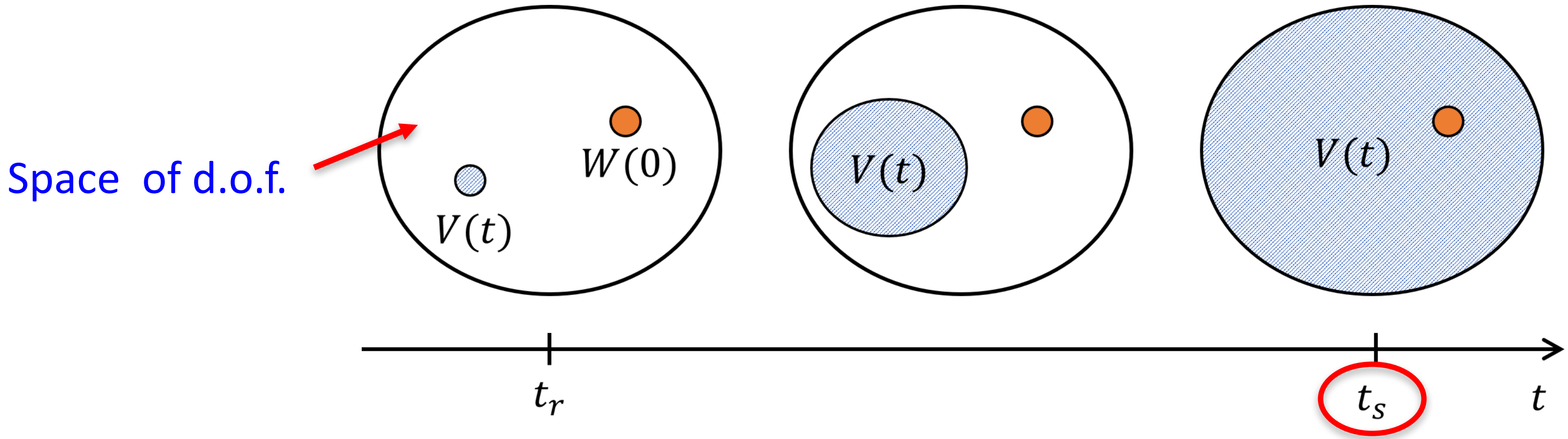
3. Some predictions and implications

Quantum many-body chaos

Scrambling of quantum information



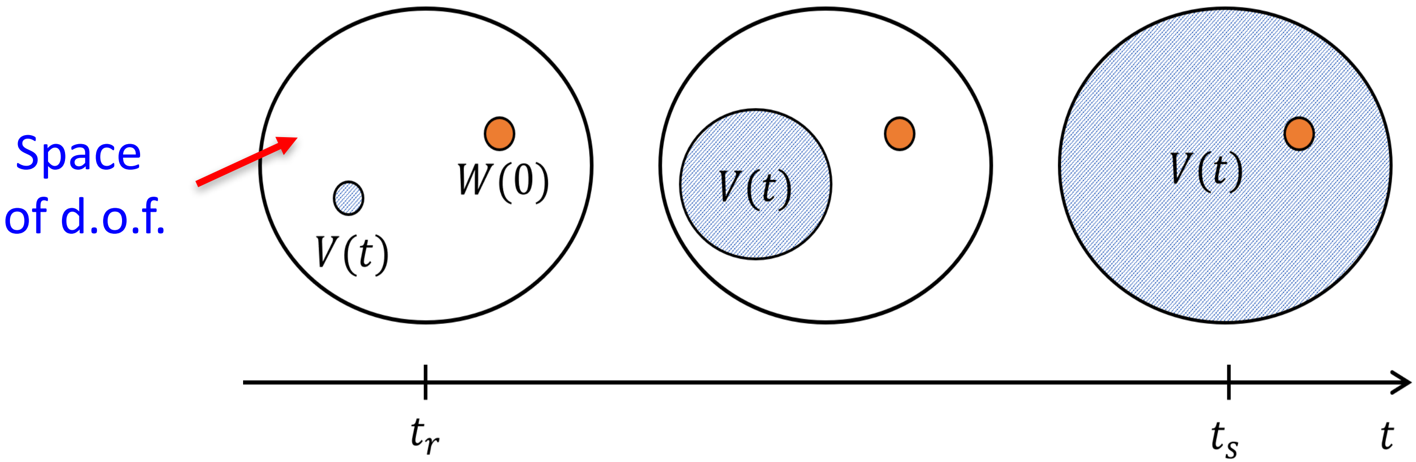
$$V(t) = e^{iHt}V(0)e^{-iHt}$$



Remarkable university!

(scrambling time)

Scrambling and quantum chaos



$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0}$$

$$\sim \frac{1}{N} e^{\lambda t}$$ (chaotic systems)

V, W generic
few-body operators

Larkin and Ovchinnikov, 1969,
Shenker, Stanford, Kitaev 2013

quantum
Chaos bound:

$$\lambda \leq \frac{2\pi}{\beta_0 \hbar}$$

Maldacena, Shenker,
Stanford, 2015

Ballistic spreading

$$C(t,\vec{x}) \equiv -\langle [V(t,\vec{x}), W(0)]^2 \rangle_{\beta_0}$$

$$\sim \frac{1}{\mathcal{N}} e^{\lambda (t - \frac{|\vec{x}|}{v_B})}$$

holographic theories,
SYK chain

Roberts, Shenker, Stanford,
Susskind, Gu, Qi, …

………

v_B : Butterfly velocity

Maximally chaotic systems

$$\lambda_{\max} = \frac{2\pi}{\beta_0 \hbar}$$

1. all holographic systems in the classical gravity limit

2. Sachdev-Ye-Kitaev (SYK) model and its variants in the low temperature limit

3. two-dimensional CFTs in the large central charge limit

....

What is special about maximally chaotic systems?

A common theme:

finite temperature dynamics of stress tensor
plays leading role

Conjecture:

In a maximally chaotic system, universal features of chaos can be captured by an effective theory governing finite temperature dynamics of stress tensor (i.e. hydrodynamics).

Connections between quantum many-body chaos and transports?

- Quantum chaos operates outside the standard hydro regime.

$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0} \sim \frac{1}{\mathcal{N}} e^{\lambda t}$$

$$\delta t \sim \frac{1}{\lambda} \qquad \lambda \sim \frac{1}{\tau_{\rm relax}} \sim \frac{T}{\hbar} \qquad \boxed{\delta t \sim \tau_{\rm relax}}$$

This is quantum chaos, different regime from turbulence.

- Are standard hydrodynamic variables still meaningful at such scales?

- Need both statistical and quantum fluctuations

Thus we need a theory to capture quantum dynamics of stress tensor ("quantum" hydrodynamics) which is applicable at much shorter spacetime scales than those of conventional hydrodynamics.

To describe chaos, in such a theory exponential growth must come side by side with energy-momentum conservation?

Quantum hydrodynamics
for quantum many-body chaos

Will use a new formulation of hydrodynamics we developed recently which is based on symmetries and action principle:

arXiv: 1511.03646, 1612.07705, 1701.07817 Crossley, Glorioso, HL

a review: 1805.09331 Paolo Glorioso, HL

See also

Dubovsky, T. Gregoire, A. Nicolis and R. Rattazzi

Grozdanov and J. Polonyi Kovtun, G. D. Moore and P. Romatschke

Haehl, R. Loganayagam and M. Rangamani

Jensen, Pinzani-Fokeeva and Yarom

Standard formulation of hydrodynamics:

Phenomenological equations for transports of conserved quantities at large spacetime scales.

Hydrodynamic fluctuations: adding by hand stochastic sources (near equilibrium).

New effective action approach:

Hydrodynamics is derived as an effective field theory (EFT) describing a general quantum statistical system at large distances and times, based on symmetries and unitarity alone.

In this formalism,

based on symmetries and unitarity alone, no phenomenological inputs needed, convenient framework to derive new hydrodynamics.

Dynamical variables can in principle defined at any scales

derivative expansion does not play a fundamental role

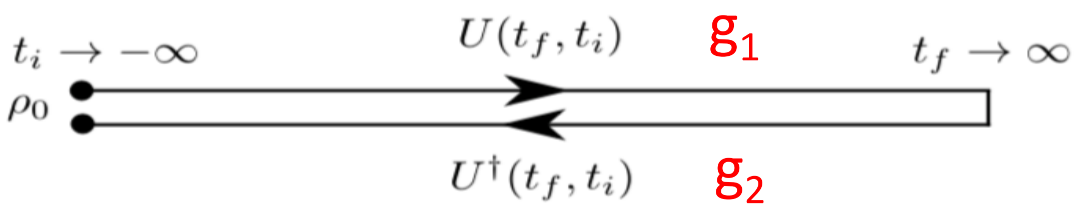
applicable to quantum level

treat systematically both quantum and statistical fluctuations in far-from-equilibrium situations

Dynamical variables

Start with a reformulation of conservation laws.

Trick: put the system in a curved spacetime: because of energy-momentum conservation, the system should be diffeomorphism invariant



$$W[g_1, g_2] = W[\tilde{g}_1, \tilde{g}_2] \qquad \tilde{g}_{1\mu\nu}(x) = \frac{\partial y_1^\sigma}{\partial x^\mu} g_{1\sigma\rho}(y_1(x)) \frac{\partial y_1^\rho}{\partial x^\nu}$$

Conservation laws = diffeomorphism invariances

Promote spacetime coordinates into dynamical variables

$$x^\mu \to X^\mu(\sigma^a)$$

σ^a : a fixed set of spacetime coordinates

→ Equations of X^μ equivalent to energy-momentum conservation.

Hydrodynamics: a theory of dynamical coordinate transformations.

σ^i : label individual fluid elements, σ^0 : internal time

Recovered Lagrangian description of fluids!

Action for hydrodynamics:

Most general action of $X_1^\mu(\sigma^a), X_2^\mu(\sigma^a)$ satisfying various symmetries

Symmetries

1. Symmetries defining a fluid:

$$\sigma^i \to \sigma'^i(\sigma^i), \quad \sigma^0 \to \sigma^0$$

$$\sigma^0 \to \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \to \sigma^i$$

2. Constraints from quantum unitarity (survive in the classical limit)

3. A Z_2 symmetry: dynamical KMS symmetry, which imposes micro-time-reversibility and local equilibrium

→ A "statistical" field theory which fully recovers the standard hydrodynamics as equations of motion, including all phenomenological constraints.

An example

Lagrangian for Navier-Stokes equation in two-dimension:

$$\mathcal{L} = \frac{1}{2\nu}(D_t\phi)^2 + \frac{\nu}{2}(\nabla^2\phi)^2$$

$$D_t\phi = \partial_t\phi + \epsilon_{ij}\frac{1}{\nabla^2}\partial_i\nabla^2\phi\partial_j\phi$$

$$v_i = \epsilon_{ij}\partial_j\phi \qquad \nu : \text{viscosity}$$

This Lagrangian can be written on phenomenological ground near equilibrium, but in fact applies to far-from-equilibrium.

Chaos EFT from quantum hydrodynamics

Need two additional inputs inferred from known maximally chaotic systems

Mike Blake, Hyunseok Lee, HL, arXiv: 1801.00010 (JHEP)

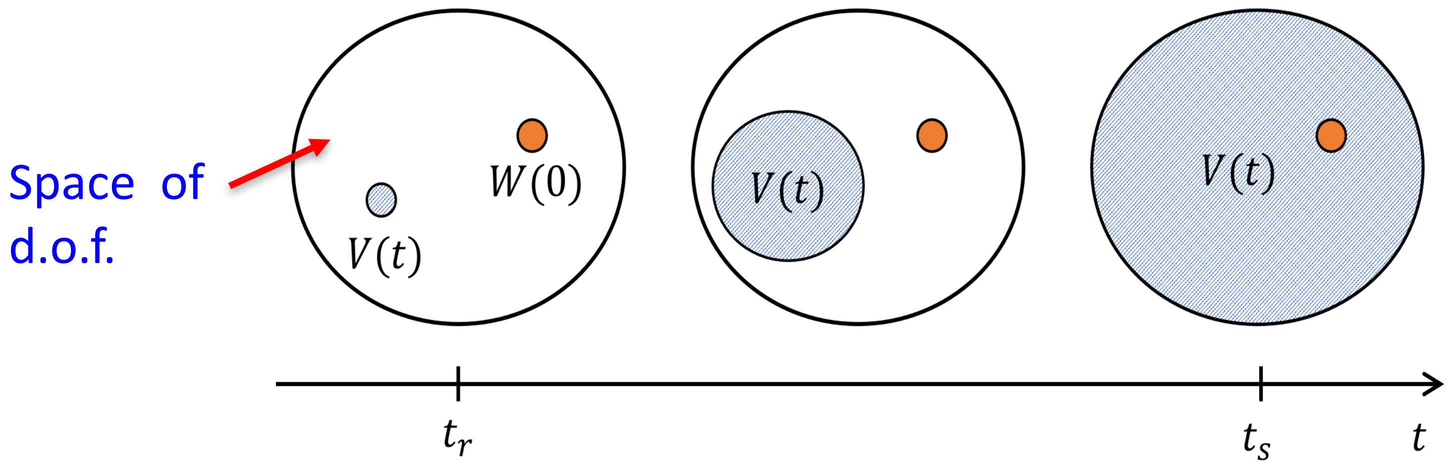
Mike Blake, Richard Davison, Saso Grozdanov, HL, 1809.01169 (JHEP)

Mike Blake, HL, to appear

As an illustration, consider a system with only energy conservation.

Dynamical variable: $\sigma(t, \vec{x})$, $X_a(t, \vec{x})$

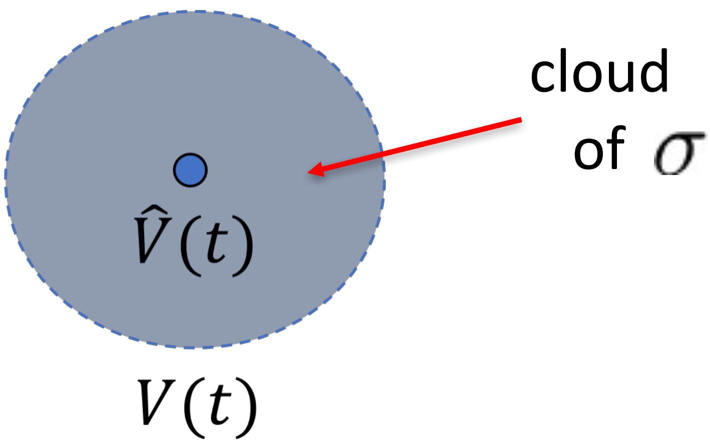
Quantum hydrodynamics for chaos (I)



Scrambling can be described by the field associated with energy conservation (i.e. associated with time diff).

dressing function

$$V(t) = f\left(\hat{V}(t), \sigma\right) =$$



Quantum hydrodynamics for chaos (II)

a chaotic system with Lyapunov exponent λ has an emergent shift symmetry:

$$e^{-\lambda\sigma} \to e^{-\lambda\sigma} + a$$

Equilibrium configuration: $\sigma = t$

$$e^{-\lambda\sigma(t)} = e^{-\lambda t} + a$$

"shock wave solution"

Chaos EFT: Most general quantum hydro theory with the shift symmetry (no derivative expansion)

Schwarzian for SYK is a special example of such a theory.

The coupling of generic operator V to σ also respect the shift symmetry.

Hydrodynamics for energy conservation

Dynamical variables: $\sigma(t), X_a(t)$ (time reparameterization)

$$\mathcal{L}_{\rm hydro} = -H\partial_t X_a - G_i \partial_i X_a + \frac{i}{2} M_1 (\partial_t X_a)^2 + \frac{i}{2} M_2 (\partial_i X_a)^2 + O(a^3)$$

$H(\partial_t \sigma, \partial_i \sigma, \cdots)$: energy density

$G_i(\partial_t \sigma, \partial_i \sigma, \cdots)$: energy flux

Equation of motion: energy conservation

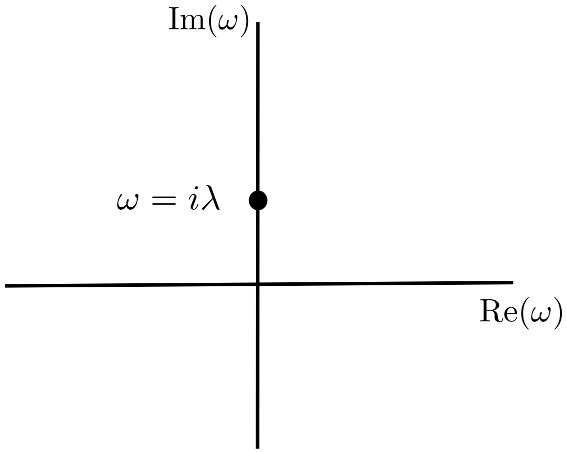
Equilibrium: $\sigma = t$ Leading order in derivatives: thermal diffusion

They include all derivative orders, nonlocal to scale β_0

Immediate consequence of the shift symmetry

$$G_R = \langle \sigma(t,\vec{x})\sigma(0)\rangle_{\beta_0}$$

$$\sim \theta(t) e^{\lambda(t-\frac{|\vec{x}|}{v_B})} + \cdots ,$$

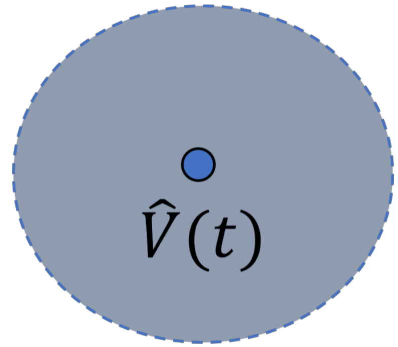


$$v_B = \#\sqrt{D\lambda}$$

D: energy diffusion constant

Four-point functions

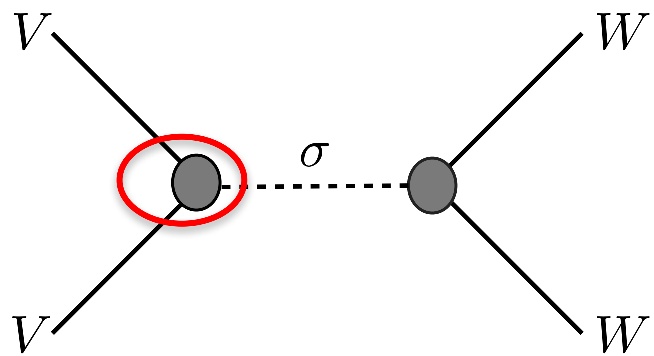
$$V(t) = f\big(\hat{V}(t), \sigma\big) =$$



$V(t)$

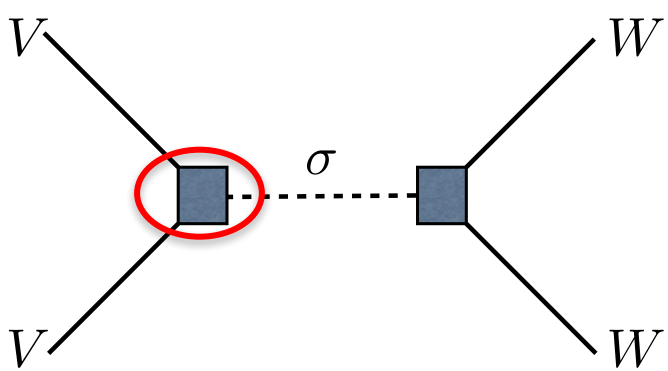
$$\langle \hat{V} \hat{W} \rangle_{\beta_0} = 0$$

TOC



No exponential growth

OTOC



$$\sim \theta(t) e^{\lambda\left(t - \frac{|\vec{x}|}{v_B}\right)} + \cdots ,$$

Energy density and energy flux are composite operators of σ

They are invariant under the shift symmetry.

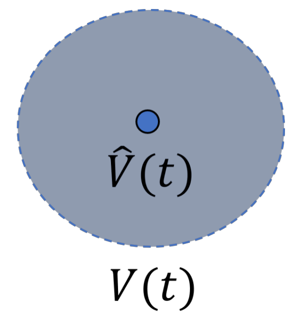
They are blind to the exponential growth of σ

Maximal chaos

By construction the quantum hydrodynamics ensures fluctuation-dissipation relations of all correlation functions of the stress tensor (for any λ).

The coupling f should also respect the shift symmetry

$$V(t) = f\left(\hat{V}(t), \sigma\right) =$$



Combine this with fluctuation-dissipation relation of two-point functions of V

$$\Rightarrow \quad \lambda = \frac{2\pi}{\beta_0 \hbar} = \lambda_{\max}$$

Implications and predictions

Phenomenon of pole skipping

The same field is responsible for both energy conservation and chaos → Predicts a new phenomenon

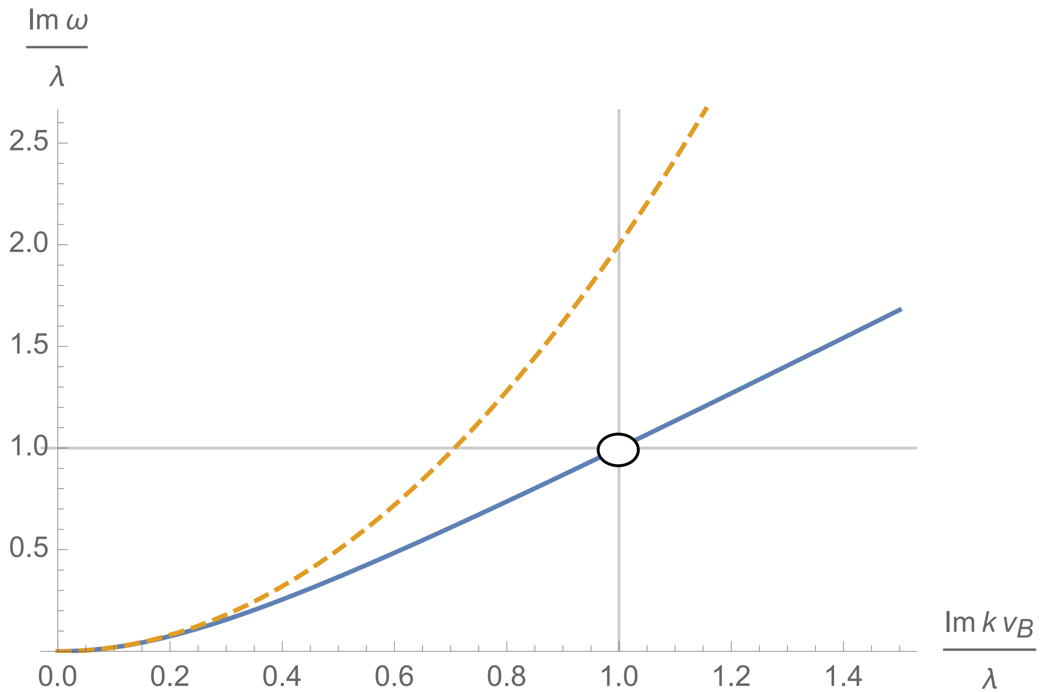
Consider energy density two-point function, which has a diffusion pole (without momentum conservation)

$$\omega = \omega(k) = -iD_E k^2 + O(k^3)$$

A line of poles in complex frequency plane when changing k

Analytic continue k to imaginary values

$$\omega = \omega(k) = -iD_E k^2 + O(k^3)$$



Predictions:

1. the pole line passes the point

$$\omega = i\lambda,\, k = \pm i\frac{\lambda}{v_B}$$

2. Precisely at that point the pole is skipped (with zero residue).

Energy density two-point function does not have a well-defined value at this point!

Pole-skipping was in fact already seen slightly earlier in holographic systems dual to a AdS_5 Schwarzschild by Grozdanov, Schalm, Scopelliti (arXiv:1710.00921), in a search for connections between hydro and chaos.

Pole-skipping can also be checked to happen in

- the SYK chain model of Gu, Qi, Stanford
- Two-dimensional CFT in the large c limit

Haehl and Rozali, 2018

pole-skipping universal in all holographic systems at a finite temperature (with or without momentum conservation).

Blake, Davison, Grozdanov, HL, 2018

$$ds^2 = -r^2 f(r) dv^2 + 2dvdr + h(r) d\vec{x}^2$$

$$\left(-i\frac{d}{2}\omega h'(r_0) + k^2\right)\delta g_{vv}^{(0)} - i(2\pi T + i\omega)\left[\omega \delta g_{x^i x^i}^{(0)} + 2k\delta g_{vx}^{(0)}\right] = 0$$

$$(d\pi T h'(r_0) + k^2)\delta g_{vv}^{(0)} = 0\,,$$ (shock wave equation)

at $\omega = i\lambda,\, k = \pm i\frac{\lambda}{v_B}$ One less equation to solve!

Also holds in higher derivative gravity systems. Grozdanov 2018

Implications and predictions (II)

$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0} = \text{TOCs} - \text{OTOCs}$$

$$\text{OTOCs} = \langle W(0)V(t)W(0)V(t) \rangle_{\beta_0} + \langle V(t)W(0)V(t)W(0) \rangle_{\beta_0}$$

An implication of chaos EFT: to leading order in large $\mathcal{N}$

$$C(t) = (ia - ia) \times e^{\lambda t} + \cdots = 0 \times e^{\lambda t}$$

Is consistent with the elastic contribution of known maximally chaotic systems.

Connection between butterfly velocity and energy diffusion

In various holographic examples and CM models the butterfly velocity appear to be related to the thermal/energy diffusion constant (Blake, Donos, Gu, Qi,Stanford ….)

From chaos EFT: energy diffusion constant

$$D = O(1) v_B^2 \tau = O(1) \frac{v_B^2}{\lambda_{\max}}, \quad \tau = \frac{2\pi}{\beta}$$

Concluding remarks

- effects of ``hydrodynamic fluctuations," i.e. $1/\mathcal{N}$ Corrections.

- For non-maximal chaos, there should be a corresponding effective theory, but the effective field only partially overlaps with the hydro fields.

Need an effective field theory of Pomeron.

- Shift symmetry plays a key role in the discussion

Very powerful: constrains hydrodynamics to all derivative orders, leads to various predictions ….

Understanding its origin should shed much light on dynamics of general quantum many-body systems.

Thank You !